Design of Cycle Optimized Interleavers for Turbo Codes

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Abstract:

In this paper the design of interleavers for turbo codes is considered. The two major criteria for the construction are the minimal distance and the passing of the extrinsic information between the decoders due to the iterative structure of the decoder. This second criterion depends on the number of short cycles in the Tanner graph of the turbo codes. The principal aim of the proposed interleaver called cycle optimized interleaver is to avoid the short cycles. We analyze this construction using cycle distribution and iterative decoding suitability. We point out that this construction is similar to the correlation design interleaver proposed by Hokfelt and als. Simulation results show that these interleavers improve the error performances compared to other constructions.

Keywords: convolutional codes, turbo codes, interleaving.

1. INTRODUCTION

Parallel concatenated convolutional codes or turbo codes are one of the most powerful error correcting codes. Their performances can be improved by optimizing the interleaver. The two major criteria for the construction of interleavers are the minimal distance of turbo codes and the passing of extrinsic information from one decoder to the other due to the iterative structure of the decoder. The second criterion must enable the iterative decoder to reach the same performances as the maximum likelihood decoder. Different researchers have proposed interleaver construction methods based on the optimization of the minimal distance but these constructions are not efficient according to the passing of extrinsic information [1][2]. In his thesis, Wiberg [3] emphasizes the importance of the cycles in turbo codes. In this paper, we will introduce an interleaver construction method which avoids short cycles in the Tanner graph of the turbo codes and as a consequence improves the passing of extrinsic information.

Section 2 discusses the passing of extrinsic information and the importance of cycles in iterative decoding. According to this, we then introduce in section 3 a new interleaver design based on cycle optimization. In section 4 we will compare this interleaver with other classical interleavers according to the error correcting performances of the associated iterative decoder.

2. ITERATIVE DECODING AND CYCLES

The structure of the iterative decoder for the classical rate $R = 1/3$ turbo codes is given in Figure 1.

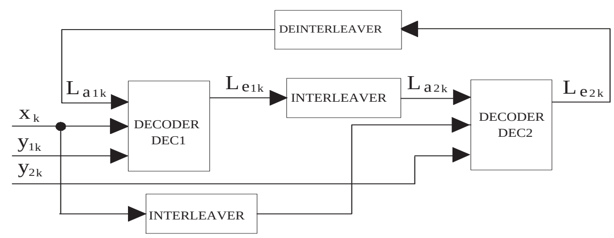


Figure 1: Iterative decoder structure

At the first iteration the MAP decoder DEC1 calculates the extrinsic output information Le_{1k} from the channel information $x_{k'}$ $(\forall k' \neq k)$ and $\mathbf{y}_1$ since $\mathbf{La}_1 = \mathbf{0}$. The correlation $\rho_{Le_{1k},x_{k'}}$ between Le_{1k} and $x_{k'}$ for $k = 50$ for constituent recursive convolutional encoder $(15, 17)_{oct}$ is given in Figure 2. According to [4], $\rho_{Le_{1k},x_{k'}}$ can be approximated using an exponentially decaying function. The effective dependency between Le_{1k} and $x_{k'}$ is function of the state space of the constituent code.

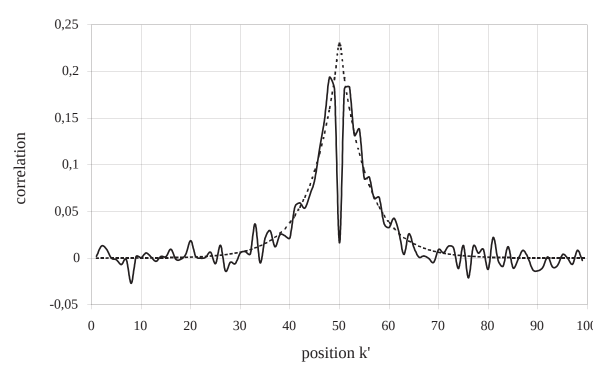


Figure 2: Correlation between extrinsic and channel information

Then the extrinsic output information Le_{2k} is calculated from channel information $x_{k'}$ $(\forall k' \neq k)$, $\mathbf{y}_2$ and a priori information $\mathbf{La}_2$. According to the second criterion, the interleaver should be chosen in order to reduce the correlation between the different information used for the calculation of the extrinsic output information. This criterion can be satisfied if the interleaver avoids the short cycles in the Tanner graph of the turbo codes.

An interleaver is described by the invertible function $\pi : \mathbb{Z} \to \mathbb{Z}$ which is a permutation on the integers $\mathbb{Z}$. A size N interleaver can also be represented using a $N \times N$ permutation matrix I.

$$I = \{a_{ij}\}_{N\times N} \quad \text{with} \quad a_{ij} \in \{0, 1\}$$

If a_{ij}=1 then the bit u_i is mapped to the bit v_j. For each pair of positions (i, j) we call primary cycle the cycle composed of the two interleaver edges $(i, \pi(i))$ and $(j, \pi(j))$ using the Tanner graph representation of turbo codes. The primary cycles are important for the passing of extrinsic information between the decoders DEC1 and DEC2 at the first iteration. For a size N interleaver, there are exactly $(N^2 - N)/2$ different primary cycles. An example of primary cycle is given in Figure 3 in bold line. We define the length of the primary cycle by $l(i, j) = |i - j| + |\pi(i) - \pi(j)|$. The minimum length cycle g of an interleaver is given by $g = \min_{i,j} l(i, j)$.

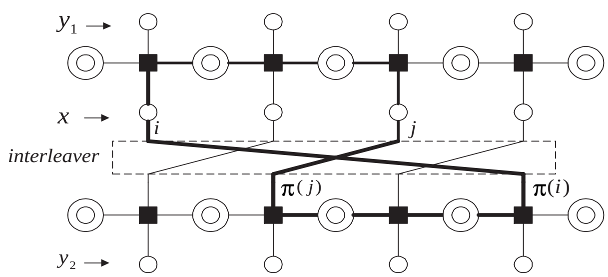


Figure 3: Example of primary cycle

3. INTERLEAVER DESIGN

A cycle optimized interleaver with parameter L is defined as follows: two bits separated by X bits $(X \leq L - 2)$ in the input sequence u should be separated with at least $L - 2 - X$ bits after interleaving. The construction is done element by element : for each position i $(1 \leq i \leq N)$, the permutated position $\pi(i)$ is chosen randomly amongst the remainder positions and tested to the $L - 2$ previous positions j and permutated positions $\pi(j)$; if $l(i, j) < L$, the permutated position $\pi(i)$ is rejected. If no more permutated positions are available, the construction is started again. Choosing $L \leq \sqrt{N} + 2$ usually gives a solution. It is shown in figure 4 that the construction of a cycle optimized interleaver is equivalent to the construction of a S-random interleaver [5] except that the forbidden windows are triangular instead of rectangular.

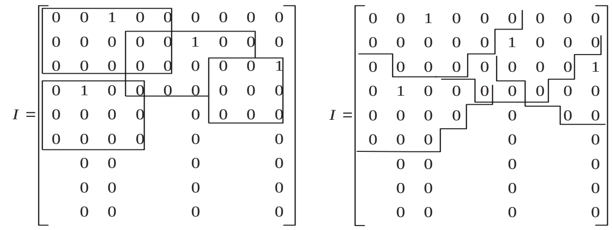
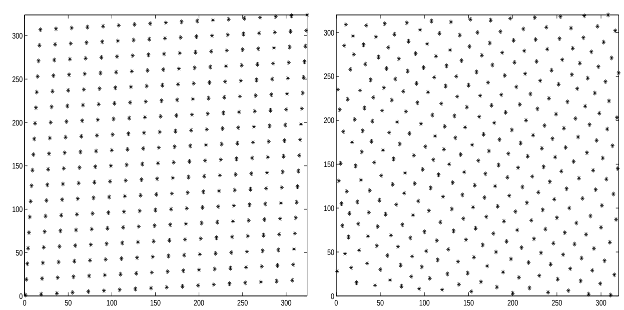


Figure 4: Comparison of a S-random $(S = 2)$ and a cycle optimized $(L = 4)$ constructions

For the cycle optimized interleaver we have $g = L = \sqrt{N} + 2$. This value is slightly greater than for a block interleaver $(g = \sqrt{N})$ and for a S-random interleaver $(g = S + 2 = \sqrt{N/2} + 2)$.

In order to reduce the number of smallest length cycles and to accelerate the construction time, we can increase the value of $L_{current}$ (typically $L_{current} = L + 10$) at the beginning of the construction and decrease it until $L = L_{current}$ when no more interleaved positions are available. The cycle optimized interleavers presented in this paper have been obtained using this refinement. It should be noticed that this improvement can also be applied for a S-random construction. The swapping method proposed in [6] can also be used to design the cycle optimized interleaver from a block interleaver.

In Figure 5, we show the permutation matrix of a block interleaver and a cycle optimized interleaver.



(a) (b)

Figure 5: (a) block interleaver $N = 324$ (b) cycle optimized interleaver $N = 320$

It can be seen that the block interleaver avoids also short primary cycles.

In [4], Hokfelt has proposed a correlation design interleaver ; the main goal of this construction is to have the extrinsic output Le_{2k} as uniformly correlated to the systematic channel information $\mathbf{x}$ as possible. He has also introduced an iterative decoding suitability (IDS) parameter to evaluate the passing of extrinsic information between the decoders during the first iteration. In table 1, we have compared the proposed interleaver construction with different interleavers of size $N = 320$ bits according to their IDS parameter. An interleaver with a low value of IDS is well suited for iterative decoding.

Table 1: Comparison of IDS parameter of different interleavers.

interleaver	IDS
Block interleaver N=324	1.08
S-random interleaver S=12	1.08
Correlated design interleaver [4]	1.00
Cycle optimized interleaver L=21	1.00

As expected, the cycle optimized interleaver and the correlated design interleaver are equivalent according to their IDS parameter. The IDS of the block interleaver is slightly higher than that of the cycle optimized interleaver and the correlated design interleaver. The difference is due to the high multiplicity of length $\sqrt{N}$ cycles of the block interleaver.

In figure 6 and 7 we have also compared three different interleavers of size $N = 320$ bits according to their cycle distributions. In [7] a probabilistic interleaver called uniform interleaver has been introduced considering all the possible interleavers. Its cycle distribution can easily be obtained as follows:

$$N_k = \sum_{m=1}^{N-1} \sum_{\substack{m+n=k \\ n \leq N-1}} \frac{(N-m)(N-n)}{N^2 - N}$$

N_k is the number of length k cycles.

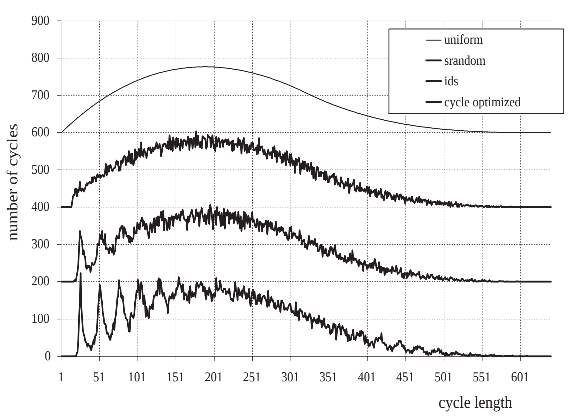


Figure 6: Cycle distributions for different interleavers

A pseudo period equal to 25 is observed on the cycle optimized interleaver. It is due to the construction which mainly allows cycles with length 25 and multiples. This property can also be observed on the correlated design interleaver.

We can verify that the minimal length cycle of the cycle optimized interleaver and correlated design interleaver is greater than the S-random interleaver.

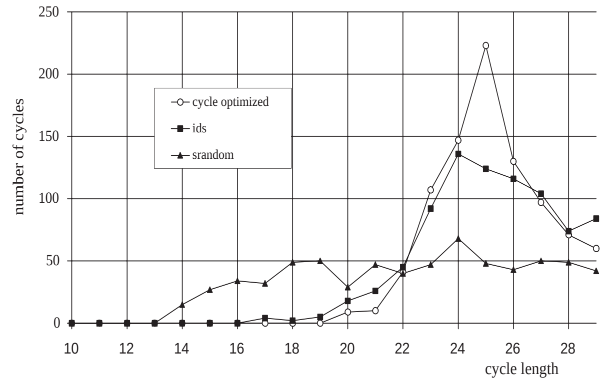


Figure 7: Cycle distributions for different interleavers

Like the correlated design method, the cycle optimized method is not efficient to fight low weight sequences. The weight 2 input sequences are the most dangerous input sequences. Assuming that the polynomial denominator of the recursive systematic code is a primitive polynomial of degree m, it is known that a weight 2 input sequence composed of 2 ones separated by $2^m - 2 + k(2^m - 1)$ zeros is a finite response input sequence. If the interleaved sequence is composed of 2 ones separated by $2^m - 2 + k'(2^m - 1)$ zeros, the global weight is low for small value of $k + k'$. To avoid these mappings during the construction of the interleaver for each i we must simply add some more forbidden positions. For small size interleavers ($N \leq 200$), it could be necessary to fight also association of weight 3 and 4 finite response input sequences. In that case a solution is to use a cost matrix for the construction of the interleaver [8]. In order to reduce the edge effect the first constituent code trellis is terminated by adding m tail bits.

4. PERFORMANCE ANALYSIS

In figure 8 we have compared the evolution of the BER in function of the number of iterations for a N=320 cycle optimized interleaver and a N=324 bloc interleaver to emphasize the influence of cycles on the efficiency of the iterative decoder. The constituent codes are recursive convolutional codes (15,17) and the signal-to-noise ratio is 2 dB. Since the minimal distance of the turbo codes with the block interleaver (26) is greater than the one with the cycle optimized interleaver (23), the expected performances at high SNR should be better using the block interleaver. After the first iteration the BER of both structures is exactly the same. We note a degradation in the performances of the iterative decoder with a block interleaver from the second iteration. This degradation is due to the secondary cycles closed during the second iteration. Indeed, the block interleaver has N length 4 secondary cycles compared to a few for a cycle optimized interleaver.

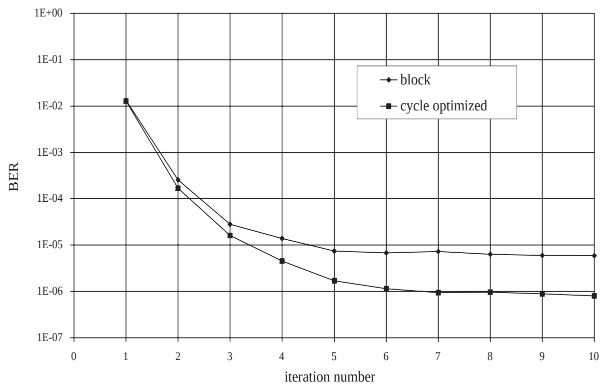


Figure 8: Evolution of BER using a block and a cycle optimized interleaver

In figure 9 we show the union bounds and the BER performances of turbo codes using $N = 320$ S-random and cycle optimized interleavers. The constituent codes are two recursive convolutional codes (15,17) and the code rate = 1/3. The performance gain is approximately 0.1 dB compared to the S-random interleaver.

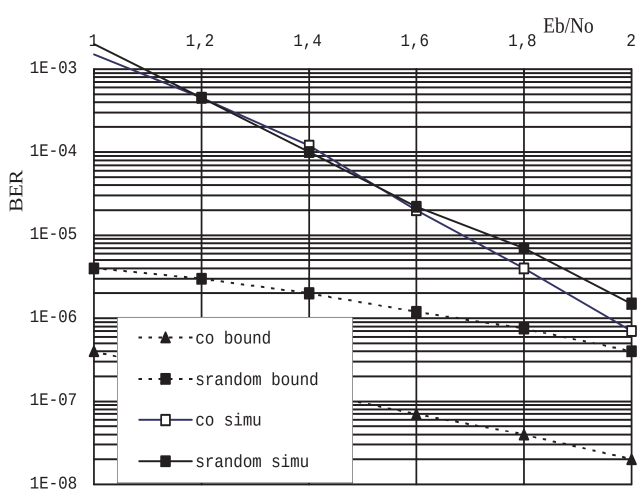


Figure 9: Performance comparison of the S-random and cycle optimized interleaver $BER = f(Eb/No)$

5. CONCLUSION

In this paper we have proposed a new interleaver called cycle optimized interleaver. We have shown that cycle optimized and constraint design interleavers are equivalent. The only difference is in the construction method: a constraint design interleaver is build based on statistical constraint while the cycle optimized interleaver construction method is geometric. The proposed method can easily be generalized to multiple turbo codes [9]. For example in the case of turbo codes with 2 interleavers, we should optimized the first interleaver (I_1) the second interleaver (I_2) and also the combination $I_1 I_2^{-1}$.

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