MLSE Receiver using the Particle Filtering over a Multipath Fading Channel

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***Abstract*— Instead of reduced-state decision-feedback sequence estimation (DFSE) equalization which is the state-of-the-art solution for the complexity reduction of the full-state Viterbi decoder over a fading multipath channel, we propose a new reduced-complexity maximum likelihood sequence detector (MLSD) based on the particle filtering technique. The computational complexity of this new detector is adapted according to the signal-to-noise ratio. Compared to the DFSE detector the particle detector offers a better trade-off between performance and computational complexity.**

I. Introduction

THE optimal detection technique of a digital signal corrupted by InterSymbol Interference (ISI) and Additive White Gaussian Noise (AWGN) is Maximum Likelihood Sequence Estimation (MLSE) if the channel parameters are perfectly known [1]. Usually, the optimal Maximum Likelihood Sequence Detector (MLSD) which minimizes the sequence error probability, is implemented using a Viterbi Decoder (VD) [2]. The main problem associated with this detector is the computational complexity of the VD. In fact, it might become quickly prohibitive in the communication systems which operate with high level modulations in long memory channels. We assume here that the dominant factor which determines the complexity of the VD is the channel memory.

Many researches have been conducted in order to reduce the computational complexity of the VD by selecting a subset of the states in the code trellis, as in the Reduced-State Sequence Estimation (RSSE) [3] and in the Decision-Feedback Sequence Estimation (DFSE) [4], or by selecting a subset of the paths in the code trellis, as in the M-algorithm [5] and in the T-algorithm [6].

In this paper, we propose an original approach to apply the particle filtering technique [7] to the detection problem in order to develop a new suboptimal MLSD. The key idea is to explore only a subset of the possible transmitted sequences with a tree-search algorithm using particles. The selected sequences of the tree are the trajectories of particles, which evolve statistically in time according to the probability that a certain symbol was transmitted conditionally to the received signal.

This paper is organized as follows. In Section II the system model is presented. The structure of the particle detector is derived in Section III. Finally, simulation results are given in Section IV.

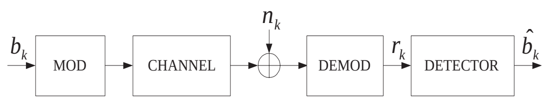

Fig. 1. Discrete-time equivalent lowpass transmission system model.

II. System Model

Fig. 1 shows the discrete-time equivalent lowpass transmission model considered in this paper. We will analyze only the binary modulation case. The generalization to more complex modulations is straightforward.

The information sequence is composed of independent and identically distributed bits. Each bit b_k is transmitted within the symbol interval of duration T. The information bits are organized into frames composed of a preamble of known bits used in the estimation of the Channel Impulse Response (CIR), a block of information bits and a tail of known bits for properly terminating the trellis. The discrete-time channel model is represented by a symbol-spaced Finite Impulse Response (FIR) filter, depicted on Fig. 2. At the detection we assume the knowledge of the CIR coefficients $\{f_{k,l}\}_{l=0}^{L}$, where L indicates the overall channel memory. Hence, the matrix model of the received signal at the input of the detector is given by:

$$\mathbf{r}_k = \mathbf{B} \cdot \mathbf{F}_k + \mathbf{n}_k, \tag{1}$$

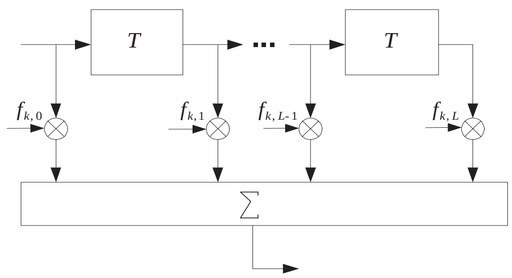

Fig. 2. Channel Model.

where:

$$\begin{aligned}
\mathbf{r}_k &= [Re\{r_k\} \;\; Im\{r_k\}], \\
\mathbf{B} &= [b_k \;\; b_{k-1} \ldots b_{k-L}] = [b_k \;\; \mathbf{B}_{k-1}], \\
\mathbf{B}_{k-1} &= [b_{k-1} \ldots b_{k-L}], \\
\mathbf{F}_k &= \begin{bmatrix} Re\{f_{k,0}\} & Im\{f_{k,0}\} \\ \vdots & \vdots \\ Re\{f_{k,L}\} & Im\{f_{k,L}\} \end{bmatrix}, \\
\mathbf{n}_k &= [Re\{n_k\} \;\; Im\{n_k\}].
\end{aligned}$$

The vector $\mathbf{n}_k$ is a discrete-time complex AWGN with zero mean, scalar variance σ_n^2 and independent real and imaginary components.

III. The particle detector

The particle filtering technique is a sequential Monte Carlo method used in non-linear/non-Gaussian tracking problems [7]. It is based upon point mass or particle representations of probability densities, which can be applied to any state space model and which generalize the traditional Kalman filtering methods. In this paper, we apply the particle filtering approach to the detection problem, in order to approximate the a posteriori probability density of the information symbols with particles. With this intention, we develop a MLSD which estimates the sequence $B_1^K = \{b_k\}_{k=1}^K$ of information bits, where K is the number of bits, maximizing the following probability:

$$\hat{B}_1^K = \arg\max_{B_1^K} p(B_1^K|R_1^K), \tag{2}$$

where $R_1^K = \{r_k\}_{k=1}^K$ indicates the sequence of received samples from time 1 to K. The particle filtering hypotheses for (1) are as follows:

- The sequence B_1^K is a Markov random process of order L:
$$p(b_k|b_1, \cdots, b_{k-1}) = p(b_k|\mathbf{B}_{k-1});$$
- The received sample at time k is independent given the transmitted bit at time k and the L precedent transmitted bits:
$$p(r_k|\mathbf{B}, A) = p(r_k|\mathbf{B}) \qquad \forall A.$$

We can determinate the time evolution of the conditional probability $p(B_1^K|R_1^K)$ in two stages:

1) **Prediction**: Calculation of the probability $p(B_1^K|R_1^{K-1})$ from the probability $p(B_1^{K-1}|R_1^{K-1})$;
2) **Correction**: Calculation of the probability $p(B_1^K|R_1^K)$ from the probability $p(B_1^K|R_1^{K-1})$.

For the first stage, applying the definition of conditional probability and considering that B_1^K is a Markov process, we can write:

$$\begin{aligned}
p(B_1^K|R_1^{K-1}) &= p(b_K|B_1^{K-1}, R_1^{K-1})p(B_1^{K-1}|R_1^{K-1}) \\
&= p(b_K|\mathbf{B}_{K-1})p(B_1^{K-1}|R_1^{K-1}).
\end{aligned} \tag{3}$$

For the second stage, using the Bayes theorem and the hypothesis of independence of the received samples, the conditional probability $p(B_1^K|R_1^K)$ can be expressed as:

$$\begin{aligned}
p(B_1^K|R_1^K) &= \frac{p(r_K|B_1^K, R_1^{K-1})p(B_1^K|R_1^{K-1})}{p(r_K|R_1^{K-1})} \\
&= \frac{p(r_K|\mathbf{B})p(B_1^K|R_1^{K-1})}{\int p(r_k|\mathbf{B})p(B_1^K|R_1^{K-1})dB_1^K}.
\end{aligned} \tag{4}$$

By substitution of (3) into (4), (4) becomes:

$$p(B_1^K|R_1^K) = \frac{p(r_K|\mathbf{B})p(b_K|\mathbf{B}_{K-1})p(B_1^{K-1}|R_1^{K-1})}{\int p(r_K|\mathbf{B})p(b_K|\mathbf{B}_{K-1})p(B_1^{K-1}|R_1^{K-1})dB_1^K}. \tag{5}$$

From (5), we can derive the classical result of the particle filtering technique. The conditional probability $p(B_1^K|R_1^K)$ is approximated by particles, characterized by a support b_k evolving in time according to the transition probability $p(b_K|\mathbf{B}_{K-1})$ and a weight depending on the probability $p(r_K|\mathbf{B})$ [7].

In this paper, we consider an original way to approximate the conditional probability $p(B_1^K|R_1^K)$ based on the particle filtering method named *conditional drawing* [8]. We observe that using the Bayes theorem and the hypothesis of independence of the received samples, we can write:

$$p(r_K|\mathbf{B}_{K-1})p(b_K|\mathbf{B}_{K-1}, r_K) = p(r_K|\mathbf{B})p(b_K|\mathbf{B}_{K-1}). \tag{6}$$

Therefore, (5) is equivalent to:

$$p(B_1^K|R_1^K) = \frac{p(r_K|\mathbf{B}_{K-1})p(b_K|\mathbf{B}_{K-1}, r_K)p(B_1^{K-1}|R_1^{K-1})}{\int p(r_K|\mathbf{B}_{K-1})p(b_K|\mathbf{B}_{K-1}, r_K)p(B_1^{K-1}|R_1^{K-1})dB_1^K}. \tag{7}$$

Analogically to the classical result, we can approximate the conditional probability $p(B_1^K|R_1^K)$ with particles, which evolve in time according to the conditional transition probability $p(b_K|\mathbf{B}_{K-1}, r_K)$ and have a weight depending on the probability $p(r_K|\mathbf{B}_{K-1})$. First, we will calculate the conditional transition probability $p(b_K|\mathbf{B}_{K-1}, r_K)$ and then the weight of a particle.

In the binary modulation case, the support of the particles can assume the values $b_K = 1$ or $b_K = -1$ and hence, we must determine the two conditional transition probabilities $p(b_K = 1|\mathbf{B}_{K-1}, r_K)$ and $p(b_K = -1|\mathbf{B}_{K-1}, r_K)$. For simplicity, we consider only the probability for the bit $b_K = 1$. Applying the Bayes theorem, the conditional transition probability is given by:

$$\begin{aligned}
&p(b_K = 1|\mathbf{B}_{K-1}, r_K) \\
&= \frac{p(r_K|b_K = 1, \mathbf{B}_{K-1})p(b_K = 1|\mathbf{B}_{K-1})}{p(r_K|\mathbf{B}_{K-1})} \\
&= \frac{p(r_K|\mathbf{B}_+)p(b_K = 1|\mathbf{B}_{K-1})}{\int p(r_K|\mathbf{B})p(b_K|\mathbf{B}_{K-1})db_K},
\end{aligned} \tag{8}$$

where we have defined $\mathbf{B}_+ = [b_K = 1 \quad \mathbf{B}_{K-1}]$ and similarly $\mathbf{B}_- = [b_K = -1 \quad \mathbf{B}_{K-1}]$. Owing to the independence and the equally probability of the information bits, we can write:

$$p(b_K = 1|\mathbf{B}_{K-1}) = p(b_K = -1|\mathbf{B}_{K-1}) = \frac{1}{2}. \tag{9}$$

Therefore, (8) becomes:

$$p(b_K = 1|\mathbf{B}_{K-1}, r_K) = \frac{p(r_K|\mathbf{B}_+)}{p(r_K|\mathbf{B}_+) + p(r_K|\mathbf{B}_-)}. \tag{10}$$

Inspecting (1), we observe that the probability densities $p(r_K|\mathbf{B}_+)$ and $p(r_K|\mathbf{B}_-)$ are Gaussian with zero mean and variance σ_n^2:

$$p(r_K|\mathbf{B}) = \frac{1}{2\pi\sqrt{\sigma_n^2}} \exp\left\{-\frac{(\mathbf{r}_K - \mathbf{B}\mathbf{F}_K)(\mathbf{r}_K - \mathbf{B}\mathbf{F}_K)^T}{2\sigma_n^2}\right\}, \tag{11}$$

where T is the transposition operator.

In order to calculate the weight of a particle, we rewrite (7) from time $K-1$ to 1:

$$p(B_1^K|R_1^K) = \frac{p(r_K|\mathbf{B}_{K-1}) \cdot\cdot p(r_1|\mathbf{B}_0) p(b_K|\mathbf{B}_{K-1}, r_K) \cdot\cdot p(b_1|\mathbf{B}_0, r_1)}{\int p(r_K|\mathbf{B}_{K-1}) \cdot\cdot p(r_1|\mathbf{B}_0) p(b_K|\mathbf{B}_{K-1}, r_K) \cdot\cdot p(b_1|\mathbf{B}_0, r_1) dB_1^K}. \tag{12}$$

We use the particle approximation for the terms related to the particle evolution:

$$p(b_K|\mathbf{B}_{K-1}, r_K) \dots p(b_1|\mathbf{B}_0, r_1) \simeq \sum_{n=1}^{N_p} \frac{1}{N_p} \delta(b_K - b_K^n) \dots \delta(b_1 - b_1^n), \tag{13}$$

where N_p is the number of particles and b_k^i is the support of the particle i at time k which can assume the values 1 or -1. We notice that the terms related to the particle evolution are taken into account in the conditional transition probability calculation and consequently, don't influence the weight determination. By substitution of (13) into (12), (12) becomes:

$$p(B_1^K|R_1^K) \simeq \sum_{n=1}^{N_p} \frac{p(r_K|\mathbf{B}_{K-1}^n) \cdot \cdot p(r_1|\mathbf{B}_0^n)}{\sum_{m=1}^{N_p} p(r_K|\mathbf{B}_{K-1}^m) \cdot \cdot p(r_1|\mathbf{B}_0^m)} \delta(b_K - b_K^n) \cdot\cdot \delta(b_1 - b_1^n), \tag{14}$$

where the term multiplying the Dirac delta represents the weight of the particle n at time K. This weight can be calculated recursively with the expression:

$$\rho_k^n = \frac{\rho_{k-1}^n p(r_k|\mathbf{B}_{k-1}^n)}{\sum_{m=1}^{N_p} \rho_{k-1}^m p(r_k|\mathbf{B}_{k-1}^m)}, \tag{15}$$

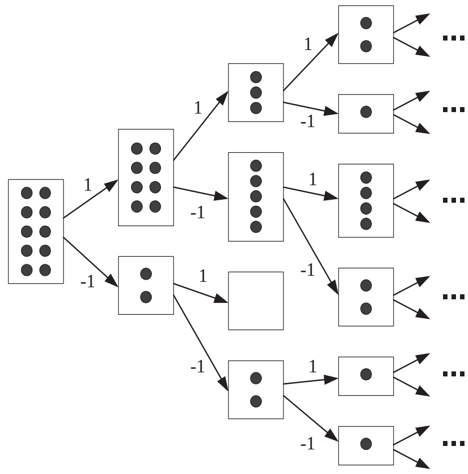

Fig. 3. Tree of the particle MLSD.

for $k = 1, \dots, K$ with the initial condition $\rho_0^n = 1/N_p$ for $n = 1, \dots, N_p$. In (15), the probability $p(r_k|\mathbf{B}_{k-1}^n)$ can be determined in the following way:

$$\begin{aligned} p(r_k|\mathbf{B}_{k-1}^n) &= \int p(r_k|b_k^n, \mathbf{B}_{k-1}^n) p(b_k^n|\mathbf{B}_{k-1}^n) db_k^n \\ &= \frac{1}{2} p(r_k|\mathbf{B}_+^n) + \frac{1}{2} p(r_k|\mathbf{B}_-^n). \end{aligned} \tag{16}$$

In order to develop a particle MLSD based on the conditional drawing technique, we use a tree-search algorithm to explore the space of the possible transmitted sequences. The root of the tree consists in a group holding all the N_p particles. Initially, the particles are equally weighted. The first bit to be estimated can assume the values 1 or -1 and hence, the particles divide in two groups proportionally to the conditional transition probabilities $p(b_1 = 1|\mathbf{B}_0, r_1)$ and $p(b_1 = -1|\mathbf{B}_0, r_1)$, where $\mathbf{B}_0$ corresponds to the last L bits of the preamble sequence and consequently, represents the initial state. At the next instant the particles will be divided in four groups. So the groups of particles form the nodes of a tree. An example of a particle tree is represented in Fig. 3. We analyze in details the generic transition process from time $k-1$ to time k. For each group at time $k-1$, we calculate the conditional transition probabilities $p(b_k = 1|\mathbf{B}_{k-1}, r_k)$ and $p(b_k = -1|\mathbf{B}_{k-1}, r_k)$, given by (10). After the division of the particles for each group at time $k-1$ proportionally to the conditional transition probability, the empty groups are eliminated. For the survivor groups at time k, we calculate the weight associated with a particle according to (15). We notice that the particles in a group have the same weight and that the weight of a group is equal to the product between the number of particles in the group and the weight of a particle.

As in the VD, the decision on the information bits in the particle MLSD is carried out after the processing of $5L$ received samples. The particle detector estimates at time $k + 5L$ the bit at time k. This corresponds to the bit b_k of the maximum weight group at time $k + 5L$. At the end of the information sequence, the particle algorithm must be terminated with the L known bits of the tail sequence in order to finish in a known final state. In this closing phase, the division of the particles in groups is deterministic; the particles transfer in the group corresponding to the known transition bit. Moreover, the calculation of the weights is modified. In (16), only the probability associated to the known bit is considered, because the other is equal to zero.

In the particle detector described above, the particles are initially concentrated in one group and during the processing of each received sample, they are spread in the space of the possible transmitted sequences. The maximum degree of exploration of this space is given by the number of particles. When each group contains only one particle, some particles can explore improbable zones. In order to improve the exploration around the most probable zones, we can force a particle redistribution. The redistribution is a very critical task, because the performance strongly depends on it. For example, we can redistribute the particles every L bits: the particles in groups with a weight inferior to $1/N_p$ are moved in the group with maximum weight.

IV. Simulation results

In this section, we present simulation results depicting the performance of the proposed particle MLSD. The adopted performance measure is the Bit Error Rate (BER) versus the Signal-to-Noise Ratio (SNR) E_b/N_0, where E_b denotes the average bit energy and N_0 the unilateral power spectral density of the noise. We determine the performance for a Global System for Mobile communications (GSM) system. We assume that the receiver detects only a slot for each Time Division Multiple Access (TDMA) frame, constituted by 8 slots. A GSM slot consists in two burst of 58 information bits separated by a midamble sequence of 26 known bits. Reference [9] shows that the backward detection of the first burst and the forward detection of the second burst give approximately the same performance. Therefore, we can consider only the forward detection of the second burst. The modulation scheme corresponds to a discrete-time linearized representation of a Gaussian Minimum Shift Keying (GMSK) signal [10], in order to simplify the structure of the demodulator. By means of a shift phase, the received signal at the input of the detector is described by (1). For the generation of the channel coefficients, perfectly known by the receiver, we consider two models. In the first model, the channel memory L is equal to 7 and the channel coefficients are given by:

$$\begin{aligned} Re\{f_{k,l}\} &= a_l \cos(2\pi f_{d,l} k T_s) \\ Im\{f_{k,l}\} &= a_l \sin(2\pi f_{d,l} k T_s), \end{aligned} \tag{17}$$

for $l = 0, \dots, 7$. The amplitudes are chosen in order to obtain a phase-minimal channel with unitary energy:

$$[a_0, \dots, a_7] = [0.56, 0.49, 0.42, 0.35, 0.28, 0.21, 0.14, 0.07].$$

The Doppler frequencies in Hz associated with each channel path are:

$$[f_{d,0}, \dots, f_{d,7}] = [10, 20, 30, 40, 50, 60, 70, 80].$$

The sampling period T_s is equal to the symbol interval $T = 3.69\mu s$.

For the second model, we consider the 12-tap Hilly Terrain (HT) GSM channel model, described in [11]. The coefficients $\{f_{k,l}\}_{l=0}^{L}$ are generated using a bank of time-shifted independent Rayleigh flat fading channel simulators. In each simulator, a white complex Gaussian noise passes through a digital second-order low-pass Chebyshev filter followed by a fifth-order Butterworth filter, which impart the Rayleigh Doppler spectrum. In simulations, the frequency Doppler for each path is equal to 83 Hz, which corresponds to a vehicle speed of 100 km/h for a 900 MHz GSM system. The channel memory L of a HT channel is equal to 6.

We compare the performance of the particle detector with the performance of a DFSE detector. The DFSE detector provides a complexity reduction of the receiver through the reduction of the number of the Viterbi states. The overall channel memory is considered and the terms of residual ISI are corrected in a Per-Survivor Processing (PSP) way along each survivor path. Fig. 4 shows the performance obtained for the first channel model. The gap between the performance of the full-state VD and the Particle Detector (PD) with 128 particles confirms that the PD is a suboptimum detection algorithm. The performance of the DFSE detector with 16 states and the PD with 8 and 128 particles are very close except at high E_b/N_0 where the performance of the PD is slightly better. On the other hand, if we reduce the number of states of the DFSE detector to 8 states, we can observe an error floor at 10^{-3}. The computational complexity of these different detectors for the first channel model is depicted in Fig. 5. Unlike the DFSE detector, the computational complexity of the PD is adapted according to the quality of the received signal. As a consequence, for the same performance, except at low E_b/N_0, the complexity of the PD is always lower than the DFSE detector. We can observe a similar behavior of the PD and of the DFSE detector for the HT100 channel model, as shown in Fig. 6. For this channel model, the computational complexity comparison is given in Fig. 7.

V. Conclusion

A reduced-complexity MLSD based on the particle filtering technique has been proposed and analyzed. The particles arranged in groups statistically explore the space of the possible transmitted sequences forming a tree. The number of paths examined by groups of particles depends on the quality of the received signal. In fact, for high SNR the particles remain concentrated in one group only, whereas for low SNR they divide into several groups. This means that the PD complexity is lower than that of a detector implemented using a Viterbi algorithm for a equal number of particles and states. Moreover, if we reduce strongly the computational complexity, the PD has shown

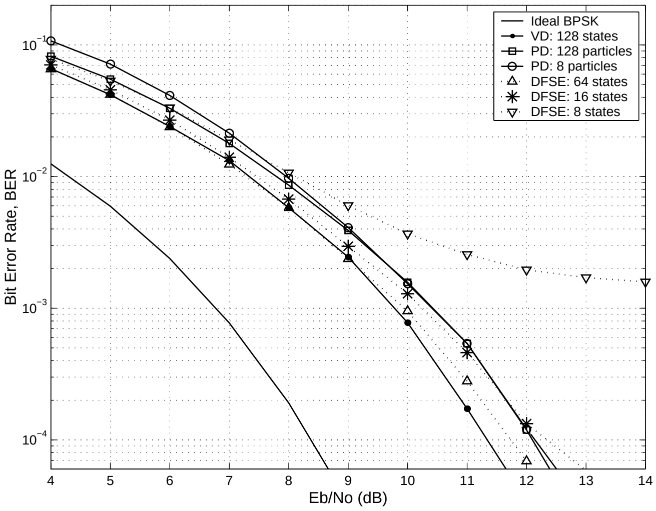

Fig. 4. BER versus E_b/N_0 for the first channel model.

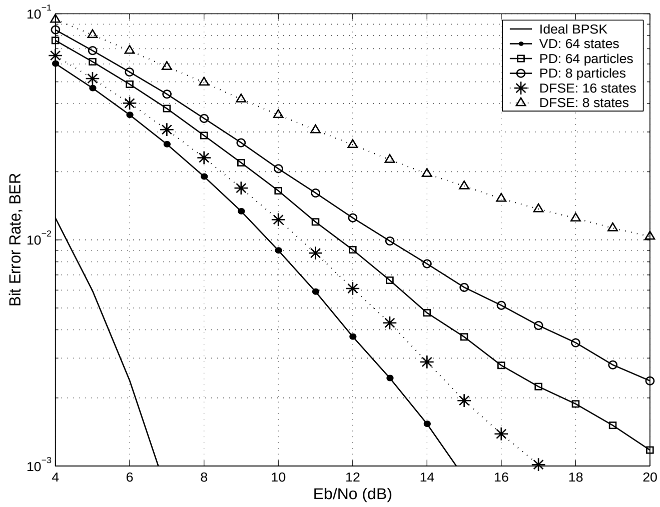

Fig. 6. BER versus E_b/N_0 for the HT100 channel.

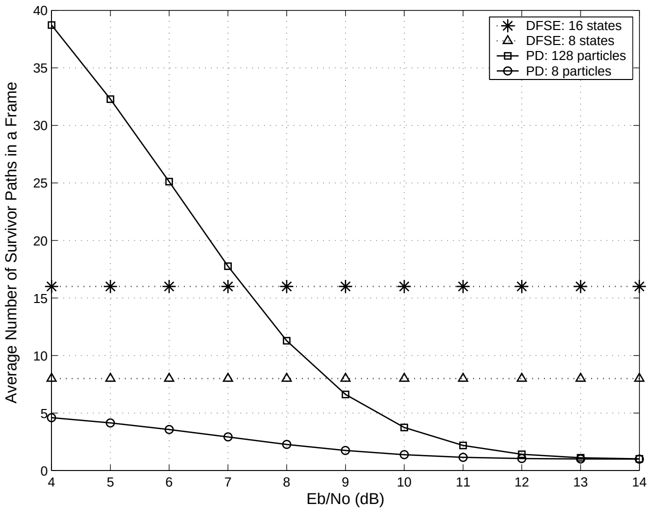

Fig. 5. Computational complexity for the first channel model.

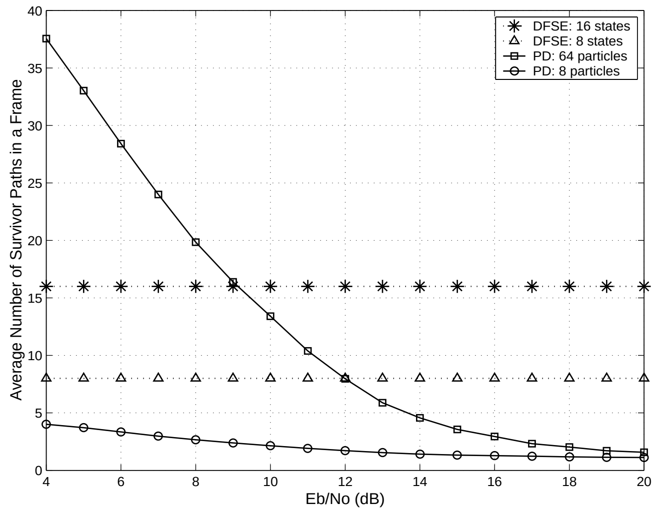

Fig. 7. Computational complexity for the HT100 channel.

better performance than that of a DFSE detector for the same number of particles and states. Hence, we can conclude that the PD represents a very good trade-off between error rate performance and computational complexity especially for communication systems operating with high level modulations and over long memory channels.

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