Abstract — In this paper, we show that recursive systematic convolutional code with primitive denominator polynomial can be decomposed. As a consequence, an RSC code can be viewed as a kind of parallel concatenation of one memory RSC code. This decomposition gives an original relation between RSC codes and turbo codes.

I. DECOMPOSITION OF RSC CODES

A recursive convolutional code is defined with a generator $G(D) = Q(D)/P(D)$, where $D$ is a unit-delay operator, $Q(D)$ and $P(D)$ are polynomials of degree $m$ over the finite field $GF(2)$:

$$Q(D) = 1 + \sum_{i=1}^{m-1} q_i D^i + D^m, \quad P(D) = 1 + \sum_{i=1}^{m-1} p_i D^i + D^m$$

The input-output equation of the encoder with input $U(D)$ and output $X(D)$ is written as $X(D) = G(D)U(D)$.

It is well known that this graph is not cycle free. In order to generate maximum length sequences of period $2^m - 1$, an all-zero polynomial $P(D)$ is chosen as a primitive polynomial. As a consequence, there exists a polynomial $M(D)$ of degree $2^m - 1 - m$ so that $M(D)P(D) = 1 + D^{2^m-1}$.

From the input-output relation, we have: [2]

$$H(D)U(D) + (1 + D^{2^m-1})X(D) = 0$$

where $H(D) = Q(D)M(D) = 1 + \sum_{i=1}^{2^m-2} h_i D^i + D^{2^m-1}$

A RSC code can be represented using a conventional Tanner graph with binary variable and parity check nodes [1]. The Tanner graph of (1) is shown in Fig. 1 for the (7,5) RSC code.

Figure 1: Tanner graph of the (7,5) RSC code from (1)

Considering the binary hidden variable $s_k = u_k + x_k$, then (1) is corresponding to the following parity-check equation for all $k$:

$$s_k = \sum_{i=1}^{2^m-2} h_i u_k(i) + s_{k-2^m+1}$$

As a consequence, the sequence of parity check bits $X_l$ is split into $2^m - 1$ different sequences $X^{(l)} = \{x^{(l)}_{j}\}, L = 1, 2, \ldots, 2^m - 1$ as shown in Fig. 2 where $g(D) = X^{(l)}U(D)/U(D)D^{-L}$. Each parity bit is described with the following state and output equations:

$$\begin{align*}
    s_{j+1}^{(l)} &= s_j^{(l)} + \sum_{i=1}^{2^m-2} h_i u_j^{(l)}(i) \\
    x_j^{(l)} &= s_j^{(l)} + u_j^{(l)}
\end{align*}$$

(2)

where $u_j^{(l)}(i) = u[j-1 + (2^m - 1)j - i]$ and $u_j^{(l)}(i = 0)$.

II. TURBO-LIKE CODES BASED ON RSC CODE DECOMPOSITION

We propose to replace each delay element $D^{-1}$ by different interleavers. This structure is a multiple parallel concatenated convolutional code or turbo-like code. Compared to classical turbo codes, the constituent codes are one memory RSC codes and consequently the decoder is much simpler. In order to avoid low weight codewords, the interleavers are build according to the constraint given by the decomposition of RSC codes. Using this principle we can derive a new class of efficient codes as recently proposed independently in [3].

REFERENCES

